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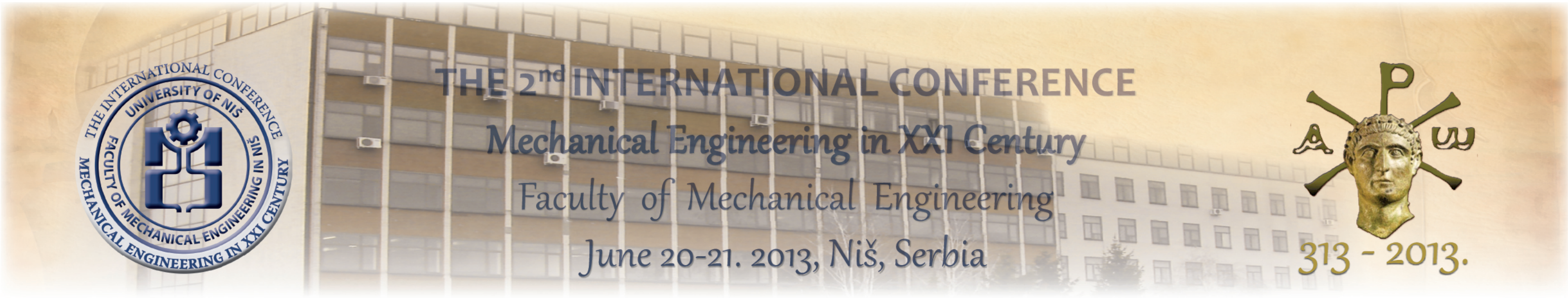

Axiszmetrical Ionized Gas Boundary Layer on a Porus Wall of The Body of Revoltion

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***Abstract*— This paper studies the ionized gas flow in the boundary layer on bodies of revolution with porous contour. The gas electroconductivity is assumed to be a function of a longitudinal coordinate x.**

Saljnikov's version of the general similarity method is used for solution of the problem. The obtained generalized boundary layer equations are solved in a four-parametric localized approximation. Based on the results, conclusions on behavior of certain physical quantities in the boundary layer have been drawn.

***Keywords*— Boundary layer, ionized gas, body of revolution, porous contour, generalized similarity method**

I. Introduction

This paper summaries results of our investigations of the ionized gas i.e. air flow in the boundary layer on bodies of revolution. Ionized gas flows in the conditions of equilibrium ionization. The contour of the body within the fluid is porous.

The primary objective of this paper is to apply the general similarity method and to solve the obtained generalized boundary layer equations.

The general similarity method was first used by Loitsianskii [1] and it was later improved by Saljnikov [2]. In its original version, it was successfully used for problems of dissociated gas flow in the boundary layer [3, 4]. Saljnikov's version of this method was applied in the temperature and MHD boundary layer theory [5, 6], and for solution of dissociated and ionized gas flow in the boundary layer [7-11]. Both versions of the general similarity method are based on usage of a momentum equation and introduction of sets of similarity parameters. In this paper, Saljnikov's version of the general similarity method is applied.

II. Mathematical Model

When aircrafts fly at supersonic speeds through the Earth's atmosphere, the temperature in the viscous boundary layer increases significantly. At high temperatures, gas (air) dissociation and ionization occur and the air becomes a multicomponent mixture of atoms, electrons and positively charged ions of oxygen, nitrogen etc. [12-14]. When the temperature in the air flow is high enough, thermochemical equilibrium is established. One of important properties of the ionized gas is its electroconductivity σ, which is a function of the temperature i.e., enthalpy [15]. If the ionized gas flows in the magnetic field of the power $B_m = B_m(x)$, an electric flow is formed in the gas. The electric flow generates Lorentz force and Joule's heat [15]. The electroconductivity is also assumed to be a function of the longitudinal coordinate x, i.e. that the electroconductivity variation law can be written as

$$\sigma = \sigma(x). \tag{1}$$

Therefore, for the case of the ionized gas flow in the magnetic field, the equations of the steady laminar boundary layer on bodies of revolution with porous wall [7, 15] take the following form:

$$\frac{\partial}{\partial x}(\rho u r^j) + \frac{\partial}{\partial y}(\rho v r^j) = 0, \quad (j=1) \tag{2}$$

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = \rho_e u_e \frac{du_e}{dx} + \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right) + \\ + \sigma B_m^2 (u_e - u), \tag{3}$$

$$\rho u \frac{\partial h}{\partial x} + \rho v \frac{\partial h}{\partial y} = -u\,\rho_e u_e \frac{du_e}{dx} + \mu\left(\frac{\partial u}{\partial y}\right)^2 + \\ + \frac{\partial}{\partial y}\left(\frac{\mu}{\Pr}\frac{\partial h}{\partial y}\right) + \sigma B_m^2 (u^2 - u\,u_e). \tag{4}$$

The corresponding boundary conditions are:

$$\begin{aligned} &u = 0, \quad v = v_w(x), \quad h = h_w = \text{const.} \quad \text{for} \quad y = 0, \\ &u \to u_e(x), \qquad\qquad h \to h_e(x) \qquad \text{for} \quad y \to \infty. \end{aligned} \tag{5}$$

In the governing mathematical model (2) is a continuity equation, (3) is dynamic and (4) is energy equation. For the terms $-\sigma B_m^2 u$ and $\sigma B_m^2 u^2$ Lorentz force and Joule's heat are determined respectively [15].

The subscript "e" stands for physical quantities at the outer edge of the boundary layer ($y \to \infty$) and the subscript "w" denotes the values on the wall of the body of revolution ($y = 0$). The given velocity $v_w(x)$, at which the gas flows perpendicularly through the porous wall of the body of revolution (Fig. 1) can be positive (at injection) or negative (at ejection).

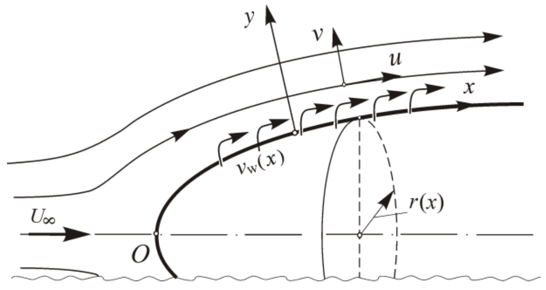

Fig.1. Ionized gas flow adjacent to the body of revolution

The continuity equation (2) can be written in a form more suitable for derivation of the momentum equation as

$$\frac{\partial}{\partial x}\left[\rho u\left(\frac{r}{L}\right)^{j}\right]+\frac{\partial}{\partial y}\left[\rho v\left(\frac{r}{L}\right)^{j}\right]=0, \qquad (L=\text{const.},\ j=1). \tag{6}$$

Here L is a constant length whose value can equal unity.

III. TRANSFORMATIONS OF THE EQUATIONS

In order to apply the general similarity method, new variables are introduced:

$$s(x)=\frac{1}{\rho_0\,\mu_0}\int_0^x \rho_{\mathrm{w}}\,\mu_{\mathrm{w}}\left(\frac{r}{L}\right)^{2j}dx, \qquad z(x,y)=\left(\frac{r}{L}\right)^{j}\int_0^y \frac{\rho}{\rho_0}\,dy, \quad (j=1). \tag{7}$$

Here, ρ_0, $\mu_0=\rho_0\nu_0$ and $\rho_{\mathrm{w}}(x)$, $\mu_{\mathrm{w}}(x)$ denote the known values of the density and dynamic, i.e., kinematic viscosity of the gas at some point of the boundary layer (subscript 0) and on the wall of the body of revolution (subscript w).

The stream function $\psi(s,z)$ is introduced using the relations:

$$u=\frac{\partial\psi}{\partial z}, \qquad \widetilde{v}=\frac{1}{(r/L)^{2j}}\frac{\rho_0\,\mu_0}{\rho_{\mathrm{w}}\,\mu_{\mathrm{w}}}\left[u\frac{\partial z}{\partial x}+v\frac{\rho}{\rho_0}\left(\frac{r}{L}\right)^{j}\right]=-\frac{\partial\psi}{\partial s} \tag{8}$$

that follow from the continuity equation (6).

Since the boundary condition for the velocity at the inner edge of the boundary layer (5) does not equal zero ($v=v_{\mathrm{w}}(x)\neq 0$), as with incompressible fluid [1], the stream function $\psi(s,z)$ is divided into two parts:

$$\psi(s,z)=\psi_{\mathrm{w}}(s)+\overline{\psi}(s,z), \qquad \overline{\psi}(s,0)=0. \tag{9}$$

Here, $\psi_{\mathrm{w}}(s)=\psi(s,0)$ stands for the stream function of the flow adjacent to the wall if the body of revolution ($z=0$).

Another change is introduced:

$$s=s\,, \quad \eta(s,z)=\frac{u_{\mathrm{e}}^{b/2}}{K(s)}z=\frac{B(s)}{\Delta^{**}(s)}z,$$

$$\overline{\psi}(s,z)=u_{\mathrm{e}}^{1-b/2}K(s)\Phi(s,\eta)=\frac{u_{\mathrm{e}}\,\Delta^{**}(s)}{B(s)}\Phi(s,\eta), \qquad h(s,z)=h_1\cdot\overline{h}(s,\eta), \quad h_1=\text{const.}, \qquad K(s)=\left(a\,\nu_0\int_0^s u_e^{b-1}ds\right)^{1/2}, \quad a,b=\text{const.} \tag{10}$$

Applying (7)-(10), the governing equations (3) and (4) are transformed into this equation system with the given boundary conditions:

$$\begin{gathered}
\frac{\partial}{\partial\eta}\left(Q\frac{\partial^2\Phi}{\partial\eta^2}\right)+\frac{aB^2+(2-b)f}{2B^2}\Phi\frac{\partial^2\Phi}{\partial\eta^2}+\frac{f}{B^2}\left[\frac{\rho_e}{\rho}-\left(\frac{\partial\Phi}{\partial\eta}\right)^2\right]+\frac{g}{B^2}\frac{\rho_e}{\rho}\left(1-\frac{\partial\Phi}{\partial\eta}\right)+\frac{\Lambda}{B}\frac{\partial^2\Phi}{\partial\eta^2}=\frac{u_e}{u_e'}\frac{f}{B^2}\left(\frac{\partial\Phi}{\partial\eta}\frac{\partial^2\Phi}{\partial s\partial\eta}-\frac{\partial\Phi}{\partial s}\frac{\partial^2\Phi}{\partial\eta^2}\right),\\
\frac{\partial}{\partial\eta}\left(\frac{Q}{\Pr}\frac{\partial\overline{h}}{\partial\eta}\right)+\frac{aB^2+(2-b)f}{2B^2}\Phi\frac{\partial\overline{h}}{\partial\eta}-\frac{2\kappa f}{B^2}\frac{\rho_e}{\rho}\frac{\partial\Phi}{\partial\eta}+2\kappa Q\left(\frac{\partial^2\Phi}{\partial\eta^2}\right)^2+\frac{\Lambda}{B}\frac{\partial\overline{h}}{\partial\eta}+\frac{2\kappa g}{B^2}\frac{\rho_e}{\rho}\frac{\partial\Phi}{\partial\eta}\left(\frac{\partial\Phi}{\partial\eta}-1\right)=\frac{u_e}{u_e'}\frac{f}{B^2}\left(\frac{\partial\Phi}{\partial\eta}\frac{\partial\overline{h}}{\partial s}-\frac{\partial\Phi}{\partial s}\frac{\partial\overline{h}}{\partial\eta}\right);\\
\Phi(s,\eta)=0,\quad \frac{\partial\Phi}{\partial\eta}=0,\quad \overline{h}=\overline{h}_{\mathrm{w}}\quad \text{for}\quad \eta=0,\\
\frac{\partial\Phi}{\partial\eta}\to 1,\quad \overline{h}\to\overline{h}_{\mathrm{e}}(s)=1-\kappa\quad \text{for}\quad \eta\to\infty,
\end{gathered} \tag{11}$$

where prim (′) stands for a derivative per the variable s.

During the transformations of the governing equations into the system (11), the usual quantities in the boundary layer theory [3, 7] are introduced: conditional displacement thickness $\Delta^{*}(s)$, conditional momentum loss thickness $\Delta^{**}(s)$, conditional thickness $\Delta_1^{*}(s)$, nondimensional friction function $\zeta(s)$ and a characteristic boundary layer function F_{m}. These quantities are defined by the expressions:

$$\begin{gathered}
\Delta^{*}(s)=\int_0^\infty\left(\frac{\rho_{\mathrm{e}}}{\rho}-\frac{u}{u_{\mathrm{e}}}\right)dz,\\
\Delta^{**}(s)=\int_0^\infty\frac{u}{u_{\mathrm{e}}}\left(1-\frac{u}{u_{\mathrm{e}}}\right)dz,\quad H=\frac{\Delta^{*}}{\Delta^{**}},\\
\Delta_1^{*}(s)=\int_0^\infty\frac{\rho_{\mathrm{e}}}{\rho}\left(1-\frac{u}{u_{\mathrm{e}}}\right)dz,\quad H_1=\frac{\Delta_1^{*}}{\Delta^{**}},\\
\zeta(s)=\left[\frac{\partial(u/u_{\mathrm{e}})}{\partial(z/\Delta^{**})}\right]_{z=0}=B\left(\frac{\partial^2\Phi}{\partial\eta^2}\right)_{\eta=0},
\end{gathered} \tag{12}$$

$$B(s)=\int_0^{\infty}\frac{\partial \Phi}{\partial \eta}\left(1-\frac{\partial \Phi}{\partial \eta}\right)d\eta,$$

$$Z^{**}=\frac{\Delta^{**2}}{\nu_0},\quad \frac{dZ^{**}}{ds}=\frac{F_{\mathrm{m}}}{u_{\mathrm{e}}},$$

$$F_{\mathrm{m}}=2\left[\zeta-\left(2+H\right)f\right]-2gH_1-2\Lambda.$$

In the equations of the system (11), there are four parameters: basic form parameter $f(s)$, magnetic parameter $g(s)$, porosity parameter $\Lambda(s)$, and local compressibility parameter [3] $\kappa(s)$. They depend on the conditions at the outer or inner edge of the boundary layer and they are defined as:

$$\begin{aligned}
&f(s)=\frac{u'_{\mathrm{e}}\,\Delta^{**2}}{\nu_0}=u'_{\mathrm{e}}\,Z^{**}=f_1(s);\\
&g(s)=S\,Z^{**}=g_1(s),\\
&S=\frac{1}{(r/L)^{2j}}\,\frac{\rho_0\,\mu_0}{\rho_{\mathrm{w}}\,\mu_{\mathrm{w}}}\,\frac{\sigma B_{\mathrm{m}}^2}{\rho_{\mathrm{e}}},\\
&\Lambda(s)=-\frac{1}{(r/L)^{j}}\,\frac{\mu_0}{\mu_{\mathrm{w}}}\,v_{\mathrm{w}}\,\frac{\Delta^{**}}{\nu_0}=-\frac{V_{\mathrm{w}}\,\Delta^{**}}{\nu_0}=\Lambda_1(s),\\
&V_{\mathrm{w}}=\frac{1}{(r/L)^{j}}\,\frac{\mu_0}{\mu_{\mathrm{w}}}\,v_{\mathrm{w}};
\end{aligned}\tag{13}$$

where $V_{\mathrm{w}}(s)$ denotes conditional transversal velocity at the inner edge of the boundary layer. The local compressibility parameter is determined as:

$$\kappa=f_0(s)=\frac{u_{\mathrm{e}}^2}{2h_1}.\tag{14}$$

In order to bring the governing equation system into a generalized form, a new stream function Φ and nondimensional enthalpy $\bar{h}$ should be introduced through general similarity transformations as:

$$\begin{aligned}
&\psi(s,\eta)=\frac{u_{\mathrm{e}}(s)\,\Delta^{**}(s)}{B(s)}\cdot\Phi\left(\eta,\,\kappa,\,(f_{\mathrm{k}}),\,(g_{\mathrm{k}}),\,(\Lambda_{\mathrm{k}})\right),\\
&h(s,\eta)=h_1\cdot\bar{h}\left(\eta,\,\kappa,\,(f_{\mathrm{k}}),\,(g_{\mathrm{k}}),\,(\Lambda_{\mathrm{k}})\right).
\end{aligned}\tag{15}$$

In (15), (f_{k}) denotes a set of form parameters of Loitsianskii's type [1], (g_{k}) stands for a set of magnetic parameters and (Λ_{k}) denotes a set of porosity parameters of the porous wall. The introduced sets of parameters are new independent variables (instead of the variable s) and they are defined as:

$$\begin{aligned}
&f_{\mathrm{k}}(s)=u_e^{\mathrm{k}-1}u_e^{(\mathrm{k})}Z^{**^{\mathrm{k}}},\qquad g_{\mathrm{k}}(s)=u_e^{\mathrm{k}-1}S^{(\mathrm{k}-1)}Z^{**^{\mathrm{k}}},\\
&\Lambda_{\mathrm{k}}(s)=-u_e^{\mathrm{k}-1}\left(\frac{V_w}{\sqrt{\nu_0}}\right)^{(\mathrm{k}-1)}\cdot Z^{**^{\mathrm{k}-1/2}},\\
&(\mathrm{k}=1,\,2,\,3,\ldots).
\end{aligned}\tag{16}$$

Each set of parameters (16) satisfies a corresponding recurrent simple differential equation:

$$\begin{aligned}
&\frac{u_{\mathrm{e}}}{u'_{\mathrm{e}}}f_1\frac{d\kappa}{ds}=2\,\kappa\,f_1=\theta_0,\\
&\frac{u_{\mathrm{e}}}{u'_{\mathrm{e}}}f_1\frac{df_{\mathrm{k}}}{ds}=\left[(\mathrm{k}-1)f_1+\mathrm{k}F_{\mathrm{m}}\right]f_{\mathrm{k}}+f_{\mathrm{k}+1}=\theta_{\mathrm{k}},\\
&\frac{u_{\mathrm{e}}}{u'_{\mathrm{e}}}f_1\frac{dg_{\mathrm{k}}}{ds}=\left[(\mathrm{k}-1)f_1+\mathrm{k}\,F_{\mathrm{m}}\right]g_{\mathrm{k}}+g_{\mathrm{k}+1}=\gamma_{\mathrm{k}},\\
&\frac{u_{\mathrm{e}}}{u'_{\mathrm{e}}}f_1\frac{d\Lambda_{\mathrm{k}}}{ds}=\left\{(\mathrm{k}-1)f_1+\left[(2\mathrm{k}-1)/2\,\right]F_{\mathrm{m}}\right\}\Lambda_{\mathrm{k}}+\\
&+\Lambda_{\mathrm{k}+1}=\chi_{\mathrm{k}}\,,\quad (\mathrm{k}=1,2,3,\ldots).
\end{aligned}\tag{17}$$

Applying similarity transformations (15) a generalized boundary layer equation system is obtained, which in four parametric $(\kappa=f_0\neq 0,\ f_1=f\neq 0,\ g_1=g\neq 0, \Lambda_1=\Lambda\neq 0,\ f_{\mathrm{k}}=g_{\mathrm{k}}=\Lambda_{\mathrm{k}}=0\ \text{for}\ \mathrm{k}\geq 2)$ three times localized approximation $(\partial/\partial\kappa=0,\ \partial/\partial g_1=0,\ \partial/\partial\Lambda_1=0)$ has the following form:

$$\begin{aligned}
&\frac{\partial}{\partial\eta}\left(Q\frac{\partial^2\Phi}{\partial\eta^2}\right)+\frac{aB^2+(2-b)f}{2B^2}\Phi\frac{\partial^2\Phi}{\partial\eta^2}+\\
&+\frac{f}{B^2}\left[\frac{\rho_{\mathrm{e}}}{\rho}-\left(\frac{\partial\Phi}{\partial\eta}\right)^2\right]+\frac{g}{B^2}\frac{\rho_{\mathrm{e}}}{\rho}\left(1-\frac{\partial\Phi}{\partial\eta}\right)+\frac{\Lambda}{B}\frac{\partial^2\Phi}{\partial\eta^2}=\\
&=\frac{F_{\mathrm{m}}f}{B^2}\left(\frac{\partial\Phi}{\partial\eta}\frac{\partial^2\Phi}{\partial\eta\partial f}-\frac{\partial\Phi}{\partial f}\frac{\partial^2\Phi}{\partial\eta^2}\right),\\
&\frac{\partial}{\partial\eta}\left(\frac{Q}{\Pr}\frac{\partial\bar{h}}{\partial\eta}\right)+\frac{aB^2+(2-b)f}{2B^2}\Phi\frac{\partial\bar{h}}{\partial\eta}-\\
&-\frac{2\kappa f}{B^2}\frac{\rho_{\mathrm{e}}}{\rho}\frac{\partial\Phi}{\partial\eta}+2\kappa Q\left(\frac{\partial^2\Phi}{\partial\eta^2}\right)^2-\\
&-\frac{2\kappa g}{B^2}\frac{\rho_{\mathrm{e}}}{\rho}\left(1-\frac{\partial\Phi}{\partial\eta}\right)\frac{\partial\Phi}{\partial\eta}+\\
&+\frac{\Lambda}{B}\frac{\partial\bar{h}}{\partial\eta}=\frac{F_{\mathrm{m}}f}{B^2}\left(\frac{\partial\Phi}{\partial\eta}\frac{\partial\bar{h}}{\partial f}-\frac{\partial\Phi}{\partial f}\frac{\partial\bar{h}}{\partial\eta}\right);\\
&\Phi=0,\quad\frac{\partial\Phi}{\partial\eta}=0,\quad\bar{h}=\bar{h}_{\mathrm{w}}=\mathrm{const.}\quad\text{for}\quad\eta=0,\\
&\frac{\partial\Phi}{\partial\eta}\to 1,\qquad\bar{h}\to\bar{h}_{\mathrm{e}}(s)=1-\kappa\quad\text{for}\quad\eta\to\infty.\\
&(\Phi=\Phi^{(1)}(\eta,\,\kappa,\,f,\,g,\,\Lambda),\,\bar{h}=\bar{h}^{(1)}(\eta,\,\kappa,\,f,\,g,\,\Lambda));
\end{aligned}\tag{18}$$

in which the subscript 1 is left out in the first parameters and where the characteristic function F_{m} is determined by the relation (12).

The obtained system of approximate generalized equations (18) is a general mathematical model of the ionized gas flow in the boundary layer adjacent to the porous wall on bodies of revolution. Due to the performed localization, the parameters $\kappa=f_0$, g and Λ became simple parameters. Hence, the system (18) is solved for in advance given values of these parameters.

IV. NUMERICAL SOLUTION, RESULTS

For the function Q and the density ratio ρ_{e}/ρ, in the system (18), analogous to the dissociated air [3], approximate dependences are adopted:

$$Q = Q(\bar{h}) \approx \left(\bar{h}_w / \bar{h}\right)^{1/3}, \quad \rho_e / \rho \approx \bar{h} / (1-\kappa). \tag{19}$$

Since Prandtl number slightly depends on temperature [3], the equations of the system (18) are solved for a constant value of this number $\Pr = 0.712$. For constants a and b, the usual values are adopted [2]: $a = 0.4408$ and $b = 5.7140$.

The system is solved by finite differences method using passage method. A concrete numerical solution of the system (18) is performed using a programme written in FORTRAN.

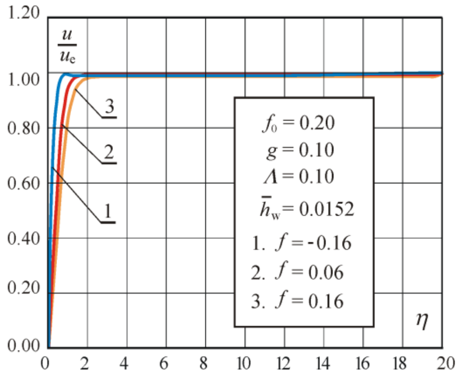

Fig.2. Diagram of the nondimensional velocity u / u_e

Only some of the obtained results are given here in the form of diagrams. Figure 2 shows the diagram of the nondimensional velocity $u / u_e = \partial \Phi / \partial \eta$ for three cross-sections of the boundary layer.

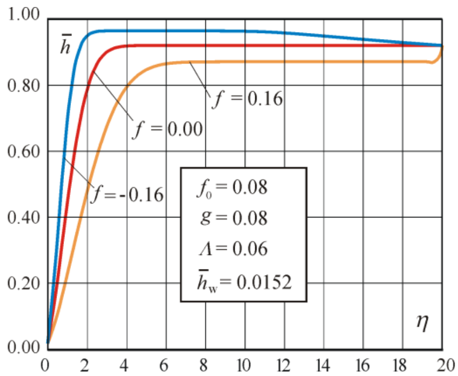

Fig.3. Distribution of the nondimensional enthalpy

The diagram in Fig. 3 presents distribution of the nondimensional enthalpy $\bar{h}$ for three cross-sections of the boundary layer.

CONCLUSIONS

This paper has shown that Saljnikov's version of the general similarity method can be successfully applied to the studied problem.

The following conclusions can be made:

- The nondimensional flow velocity u/u_e (Fig. 2) at certain cross-sections of the boundary layer on bodies of revolution converges very fast towards unity.
- The nondimensional enthalpy $\bar{h}$ converges relatively fast towards the value at the outer edge of the boundary layer.

Finally, in order to obtain more accurate results, the system (18) should be solved in a four-parametric approximation but without localization per the compressibility parameter. However, this kind of solution is fraught with difficulties, mainly of numerical nature.

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